

District Level Mathematical Olympiad[DMO]-2023/024

- Ramon wants to cut a rectangular board into square pieces (may not be identical pieces). If the board is 18 inches by 30 inches, what is the least number of square pieces he can cut without wasting any of the board?
a) 4 b) 9 c) 15 d) 6
- If $\frac{4}{5}$ of the women and $\frac{3}{4}$ of the men in a group speak english. There are 40% more men than women in the group, what fraction of the group doesn't speak English?
a) $\frac{37}{48}$ b) $\frac{11}{14}$ c) $\frac{3}{5}$ d) $\frac{11}{48}$
- The ratio of the marbles with Jack and Dave was 5:3. Now, they each have 10 more marbles with them. Which of the following cannot be the ratio of their marbles now?
a) 7:5 b) 7:4 c) 15:11 d) 15:13
- Auto lot has a 2:1 ratio of blue cars to red cars and a 6:1 ratio of red cars to orange cars on the lot. What could be the total number numbers of blue, red and orange cars on the lot?
a) 37 b) 39 c) 38 d) 40
- Kabir's college is 12 miles from his hostel. He travels 6 miles from the college to basketball practice, and then 4 miles to a computer class. If he is D miles from hostel, what is the range of possible values for D?
a) $10 \leq D \leq 12$ b) $2 \leq D \leq 16$ c) $10 \leq D \leq 22$ d) $2 \leq D \leq 22$
- The difference between maximum and minimum value of $\frac{x}{y}$ for which $(x - 2)^2 = 9$ and $(y - 3)^2 = 25$?
a) $-\frac{15}{8}$ b) $\frac{19}{8}$ c) $\frac{25}{8}$ d) $\frac{9}{8}$
- If a and b are positive integers such that $a - b = 12$ and $HCF \left\{ \frac{a^3+b^3}{a+b}, ab \right\} = 9$. Then smallest possible value of b is?
a) 2 b) 1 c) 3 d) 4
- A band has 72 members who will all be marching during halftime. They need to march in rows with the same number of students per row. If there must be between 5 and 20 students per row, in how many possible row-lengths can the band be arranged?
a) 4 b) 8 c) 7 d) 5
- The superfactorial $n\$$ is defined as $n\$ = n!^{n!^{n!^{\dots^{n!}}}}$. What is the units digit of $4\$$?
a) 0 b) 4 c) 2 d) 6
- If a and b are integers, such that $a \geq b$ When $(a+b)$ and $(2a+b)$ is divided by 10, leaves the remainder 2 and 1 respectively. Then what is the unit digit of $a - b$?
a) 6 b) 4 c) 1 d) 0
- How many different four-digit numbers can be formed by arranging the four digits in 2004?
a) 4 b) 6 c) 2 d) 8
- A bag contains 4 red marbles, 5 white marbles and 6 green marbles. A man wants to take out 3 marbles from the bag one after another without replacement. Which of the followings is the probability of getting at least 2 green marbles?
a) $\frac{44}{91}$ b) $\frac{88}{91}$ c) $\frac{11}{91}$ d) $\frac{31}{91}$
- An ant is walking on the surface of a cuboid with edges of length 3, 4 and 5. It starts walking on a vertex, and it wants to visit all the other seven vertices. It doesn't need to return to its starting vertex. What is the length of the shortest possible route that accomplishes this?
a) 24 b) 25 c) 26 d) 27

14. Three players play a game with coins. Each round the player (or one of the players) who has the largest number of coins will put one coin in a vase and then he will give each of the other players one coin. The vase is empty when the game starts and the three players possess respectively 13, 14 and 15 coins. The game ends when one of the players has lost all his coins. How many coins will be in the vase when the game ends?
a) 36 b) 37 c) 38 d) 39
15. How many positive integers a exist such that dividing 2216 by a gives remainder 29?
a) 4 b) 5 c) 6 d) 3
16. As n ranges over the positive integers, what is the maximum possible value that the greatest common divisor of $13n + 8$ and $5n + 3$ can take?
a) 3 b) 1 c) 5 d) 2
17. For what value of m the sum of the roots of the equation $2x^3 + mx^2 - 10x + 24 = 0$ is 3?
a) 3 b) 2 c) -2 d) -6
18. A girl has got five real numbers $a < b < c < d < e$. She takes the sum of each pair of numbers and writes down the ten sums. The three smallest sums are 32, 36 and 37, while the two largest sums are 48 and 51. Which of the followings is the value of e ?
a) 25.5 b) 23.5 c) 27.5 d) 26.5
19. Pick two consecutive positive integers whose sum is less than 100. Square both of those integers and then find the difference of the squares. Which of the following could be the difference?
a) 131 b) 103 c) 64 d) 79
20. A test consists of six questions worth successively 1 to 6 points. If your answer to a question is correct, the number of points that question is worth will be added to your score. If your answer is incorrect, that number of points will be deducted from your score. [eg. If you only answered questions 1, 3 and 4 correctly, your score will be $1 - 2 + 3 + 4 - 5 - 6 = -5$. What is the possible score you can get for this test?
a) 19 b) 22 c) 20 d) 18
21. A mathematician wants to approximate the width of a lake. He places 5 markers near the lake and measures the distances as shown [Fig. 1]. What is the width (in meters) of the lake from point P to point Q?
a) 150 b) 200 c) 250 d) 500
22. ABCD is a cyclic quadrilateral [Fig. 2] in which AC and BD are the diagonals. If $AB = 4.5\text{cm}$, $BC = 3.6\text{cm}$, $CD = 6.4\text{cm}$ and $AD = 5\text{cm}$, what is the value of $AC \times BD$?
a) 46.8 b) 64.8 c) 68.4 d) 48.6
23. In quadrilateral ABCD, $AD \parallel BC$ and $CD = \frac{1}{2}AB$ as [Fig. 3]. The measurement of angle B in degrees is?
a) 120 b) 135 c) 150 d) 90
24. In $\triangle ABC$ [Fig. 4] if $AD = BD = CD$ and $\angle ABD = 30^\circ$, which of the followings is the ratio of the length of sides AB, BC, and CA?
a) 1: 2: 3 b) $\sqrt{3}$: 2: 1 c) 1: 2: $\sqrt{3}$ d) 1: $\sqrt{3}$: 2

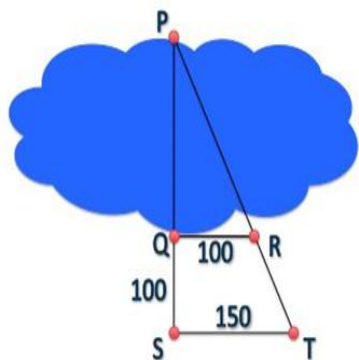


Fig. 1

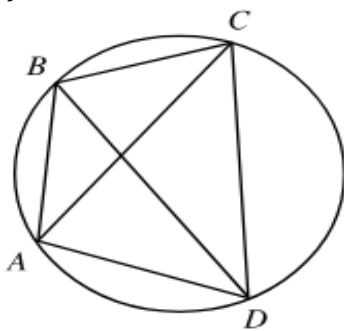


Fig. 2

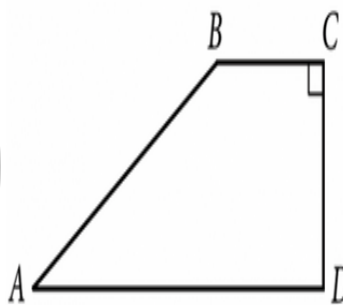


Fig. 3

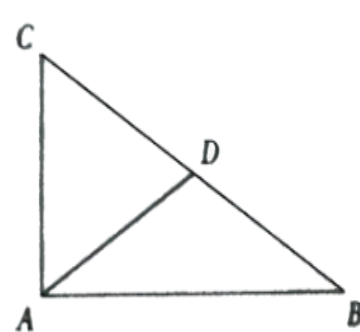


Fig. 4

25. One hundred students participate in a mathematical Olympiad. There are three problems. Problem 1 was solved by 90 participants. Problem 2 was solved by 80 participants and problem 3 has been solved by 75 participants. What is the minimal number of participants that solved all the problems?
a) 35 b) 45 c) 65 d) 55
26. A farmer has a stack of hay to feed his horse, cow, and goat. With it, he can feed his cow and horse for 12 months, or his cow and goat for 15 months, or his horse and goat for 20 months. For how many months can he feed all three of his animals?
a) $7\frac{5}{6}$ b) 9 c) 10 d) $23\frac{1}{2}$
27. The area (in sq. unit) of region bounded by the lines $y = 4x - 6$, $y = 2x - 12$ and the y.axis is?
a) 27 b) 40 c) 36 d) 72
28. A point (x, y) is randomly picked from inside the rectangle with vertices $(0,0)$, $(4,0)$, $(4,1)$, and $(0,1)$. What is the probability that $x < y$?
a) $1/4$ b) $1/6$ c) $1/16$ d) $1/8$
29. How many zero's are there at the end of $31!$?
a) 7 b) 6 c) 3 d) 8
30. A parabola with equation $y = x^2 + bx + c$ passes through the points $(-1, -11)$ and $(3, 17)$. What is c ?
a) -2 b) 4 c) -7 d) 5
31. In triangle ABC , medians AD and CE intersect at P , $PE = 1.5$, $PD = 2$, and $DE = 2.5$. What is the area of $AEDC$?
a) 13.5 b) 14 c) 15 d) 12.5
32. Evaluate $\left[\frac{3}{2}\right]^2 + \left[\left(\frac{3}{2}\right)^2\right]$. Where $[x]$ represents the ceiling function of x .
a) 4 b) 11 c) 8 d) 7
33. In how many ways can 8 people be seated around a square table with 2 people on a side? (Two configurations are considered equivalent if one is a rotation of another).
a) 10080 b) 5040 c) 28 d) 20160
34. 100 people were surveyed and asked the question: "Is fuchsia kinda pink, or purply?" Of them, 60 believe that fuchsia is "kinda pink" and 27 believe it is both "kinda pink," and also "purply." Another 17 think that fuchsia is neither "kinda pink" nor "purply." How many of those 100 people believe that fuchsia is "purply"?
a) 23 b) 33 c) 27 d) 50
35. A palindrome is a number that reads the same forwards and backwards. The sum of a particular set of three consecutive positive integers is a three-digit palindrome. If the sum is less than 220, what is the greatest possible value for the largest of the three integers in the set?
a) 67 b) 123 c) 121 d) 58
36. A book with 50 pages numbered 1 through 50 has its pages renumbered in reverse, from 50 to 1. For how many pages do both sets of page numbers share the same one's digit?
a) 10 b) 11 c) 12 d) 0
37. What is the smallest positive integer with exactly 20 positive divisors?
a) 1536 b) 432 c) 2240 d) 2520
38. What is the least positive integer n such that 80325 divides $n!$?
a) 19 b) 23 c) 13 d) 17
39. Christine must buy at least 45 fluid ounces of milk at the store. The store only sells milk in 200 milliliter bottles. If there are 33.8 fluid ounces in 1 liter, then what is the smallest number of bottles that Christine could buy?
a) 7 b) 12 c) 9 d) 5
40. The sum of the proper divisors of 18 is 21. What is the sum of the proper divisors of 198?
a) 269 b) 270 c) 468 d) 467

41. Find the smallest integer value of C such that the function $f(x) = \frac{2x^2+x+5}{x^2+4x+c}$ has a domain of all real numbers.
 a) 6 b) 4 c) 5 d) 3
42. A 6-sided die is weighted so that the probability of any number being rolled is proportional to the value of the roll. (So, for example, the probability of a 2 being rolled is twice that of a 1 being rolled.) What is the probability of rolling a 1?
 a) $1/3$ b) $1/6$ c) $1/21$ d) $1/18$
43. What is the greatest common factor (HCF) of $20!$ and 200000 ?
 a) 200000 b) 80000 c) 100000 d) 40000
44. An ice cream shop offers 6 kinds of ice cream. What is the greatest number of two scoop sundaes that can be made such that each sundae contains two types of ice cream, and no two sundaes are the same combination?
 a) 120 b) 15 c) 36 d) 24
45. A book is said to have n leaves if it is composed of n pieces of paper. On the other hand, the number of pages is twice the number of leaves because each side of a piece of paper is defined as a page. If the number of pages in a book is 3 more than a multiple of 7, and the number of leaves is greater than 100, then what is the smallest possible number of leaves?
 a) 104 b) 103 c) 203 d) 110
46. The base 5 number $34x1_5$ is divisible by 31. What is the value of x ?
 a) 6 b) 7 c) 2 d) 4
47. There are three circles and four straight lines. What is the maximum number of intersection points for these figures?
 a) 18 b) 24 c) 36 d) 21
48. In the figure given [Below Fig. 5], P and Q are the centers of two tangent circles with radii 18cm and 8cm respectively. If AB is the common tangent to both circles, what is the length of AB in cm?
 a) 24 b) 30 c) 36 d) 26
49. In the adjoining diagram, $ABCDEF$ is a regular hexagon and BDF is a triangle [Below Fig. 6]. If the area of the regular hexagon is 1 sq.cm., what is the area of the kite $BCDF$ in sq. cm ?
 a) $\frac{2}{3}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{5}{6}$ d) $\frac{3}{4}$
50. Two 4×4 squares intersect at right angles, bisecting their intersecting sides, as shown [Below Fig. 7]. The circle's diameter is the segment between the two points of intersection. What is the area of the shaded region created by removing the circle from the squares?
 a) $28 - 4\pi$ b) $18 - \pi$ c) $28 - 2\pi$ d) $18 - 4\pi$

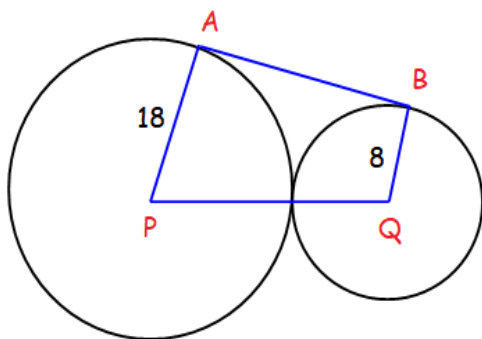


Fig . 5

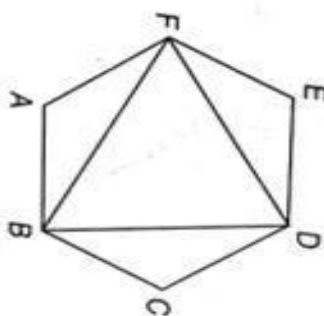


Fig. 6

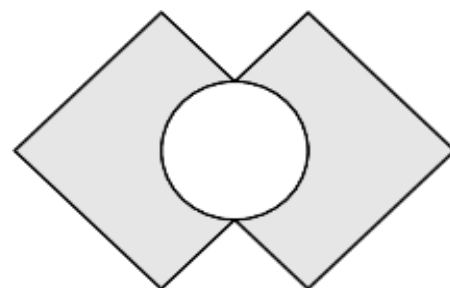


Fig. 7