

District Level Mathematical Olympiad[DMO]-2023/024

- A total of \$450 was donated to charity by 25 employees. If 15 employees donated at least \$12 but less than \$19 and 9 employees donated at least \$19, what is the maximum amount, in dollars, that the last employees could have donated?
a) \$51 b) \$99 c) \$109 d) \$89
- The price of a phone call consists of a standard connection fee, which is constant, plus a per-minute charge. A ten minute call costs Rs 2.90 and a sixteen minute call costs Rs 4.40. How much does a nineteen minutes call cost?
a) Rs 3.55 b) Rs 3.57 c) Rs 3.58 d) Rs 5.15
- $\frac{4}{5}$ of the women and $\frac{3}{4}$ of the men in a group speak english. If there are 40% more men than women in the group, what fraction of the group used to speaks non-english?
a) $\frac{37}{48}$ b) $\frac{11}{48}$ c) $\frac{3}{5}$ d) $\frac{7}{11}$
- The sum of $\sin^2 1^\circ + \sin^2 2^\circ + \dots + \sin^2 89^\circ + \sin^2 90^\circ$ is which of the following?
a) 44 b) $\frac{91}{2}$ c) 45 d) $\frac{89}{2}$
- If $k < 1$, which of the following must be negative?
a) $k^2 - 6k + 8$ b) $k^2 - 2k + 1$ c) $|k| - k^2$ d) $2^k - 3$
- If the arithmetic mean of 3, 8 and w is greater than or equal to w and smaller than or equal to $3w$, how many integer values of w exist?
a) Five b) Four c) Three d) Two
- What is the unit digit of $1! + 2! + 3! + 4! + \dots + 100!$?
a) 1 b) 2 c) 3 d) 4
- The ratio of the marbles with Jack and Dave was 13:9. Now, they each have 10 more marbles with them. Which of the following cannot be the ratio of their marbles now?
a) 7:4 b) 7:5 c) 15:11 d) 31:23
- If $\frac{1}{2} f(x) = f\left(\frac{1}{2}x\right)$, which of the following is true for all values of $f(x)$?
a) $f(x) = 2x + 2$ b) $f(x) = x^2$ c) $f(x) = \sqrt{x}$ d) $f(x) = 13x$
- Ramon wants to cut a rectangular board into square pieces. If the board is 18 inches by 30 inches, what is the least number of square pieces he can cut without wasting any of the board?
a) 4 b) 9 c) 15 d) 6
- How many whole numbers are between $\sqrt{70}$ and $\sqrt{370}$
a) 7 b) 10 c) 9 d) 11
- In a parallelogram ABCD, OA and OB are the bisectors of the angles A and B respectively at O, then an angle AOB equals to ?
a) 60° b) 75° c) 90° d) 120°
- For which of the following positive integers is the square of that integer divided by the cube root of the same integer equal to nine times that an integer?
a) 1 b) 8 c) 64 d) 27

27. The equation of the image of the line $2x + 3y = 6$ under the reflection by y-axis is
a) $2x - 3y = 6$ b) $2x - 3y + 6 = 0$ c) $2x + 3y + 6 = 0$ d) $2x + 3y = 6$
28. If $x^2 - \frac{1}{x^2} = 5 - \sqrt{3}$, then what is the value of $x^2 + \frac{1}{x^2}$?
a) $5 + \sqrt{3}$ b) $7 + \sqrt{3}$ c) $\sqrt{5 + \sqrt{3}}$ d) $\sqrt{32 - 10\sqrt{3}}$
29. During a company's first full year in business, the price of oil increased 50%. In the second year of business, the price of oil went down 50% from its price at the beginning of that year. Its price at the end of the second year is what percent of its price at the beginning of the first year?
a) 75% b) 50% c) 0% d) 25%
30. Mr. Davidson's 8th grade class has 7 more male students than female students. If $\frac{2}{3}$ of the students are male, how many students are in the class?
a) 27 b) 21 c) 18 d) 30
31. How many integers 1-9 are divisors of the five-digit number 24,516?
a) 5 b) 6 c) 8 d) 4
32. What is the sum of all of the positive factors of 360?
a) 960 b) 1170 c) 1055 d) 1230
33. Cameron writes down the smallest positive multiple of 20 that is a perfect square, the smallest positive multiple of 20 that is a perfect cube, and all the multiples of 20 between them. How many integers are in Cameron's list?
a) 45 b) 46 c) 47 d) 48
34. What is the largest divisor of 342 that is also a factor of 285?
a) 114 b) 38 c) 47 d) 57
35. If a and b are positive integers such that $a - b = 6$ and $HCF\left\{\frac{a^3+b^3}{a+b}, ab\right\} = 9$. Then the smallest possible value of b is?
a) 2 b) 1 c) 3 d) 4
36. The ones digit of the product of four consecutive positive integers is 4. If this product is greater than 1000, what is the sum of the four smallest such integers?
a) 30 b) 31 c) 32 d) 34
37. How many integers n are there such that $3 \leq n \leq 10$ and 121_n (the number written as 121 in base n) is a perfect square?
a) 4 b) 2 c) 8 d) 6
38. A band has 72 members who will all be marching during halftime. They need to march in rows with the same number of students per row. If there must be between 5 and 20 students per row, in how many possible row-lengths can the band be arranged?
a) 4 b) 8 c) 7 d) 5
39. What is the smallest positive value of x such that $(x + 5678)$ results in a palindrome?
a) 44327 b) 207 c) 988 d) 97
40. The superfactorial $n\$$ is defined as $n\$ = n!^{n!^{n!^{\dots n!}}}$. What is the units digit of $4\$$?
a) 6 b) 4 c) 2 d) 0
41. Let A be the product of the divisors of 42. How many prime factors does A have?
a) 4 b) 5 c) 7 d) 3

42. How many positive integers t will make the expression $\frac{19}{t} + \frac{5}{t}$ have an integer value?
 a) 6 b) 8 c) 4 d) 12
43. What is the remainder when 1999^{2000} is divided by 5?
 a) 0 b) 1 c) 2 d) 3
44. For how many ordered pairs of positive integers (x,y) is $x+2y = 100$?
 a) 45 b) 48 c) 49 d) 50
45. If a and b are integers, such that $a \geq b$. When $(a+b)$ and $(2a+b)$ is divided by 10, leaves the remainder 2 and 1 respectively. Then what is the unit digit of $a - b$?
 a) 6 b) 4 c) 1 d) 0
46. Suzanne walks four miles every third day. What is the fewest number of miles she can walk in February?
 a) 40 b) 44 c) 36 d) 32
47. How many different four-digit numbers can be formed by arranging the four digits in 2004?
 a) 6 b) 4 c) 2 d) 8
48. Two siblings, Andy and Alexa, bake 24 cookies. Alexa eats some number of cookies that is a positive multiple of the number of cookies eaten by her brother. If the siblings finish all 24 cookies, then what is the maximum number of cookies that the brother, Andy, could have eaten?
 a) 6 b) 8 c) 12 d) 15
49. From figure below (**Fig. 1**), what is the ratio of the volume of the cone to the volume of the cylinder?
 a) $\frac{1}{6}$ b) $\frac{1}{3}$ c) $\frac{2}{3}$ d) $\frac{1}{4}$
50. In the figure below (**Fig. 2**), the largest circle has a radius of six meters. Five congruent smaller circles are placed as shown and are lined up in east-to-west and north-to-south orientations. What is the radius in meters of one of the five smaller circles?
 a) 3 b) 1.5 c) 2.5 d) 2

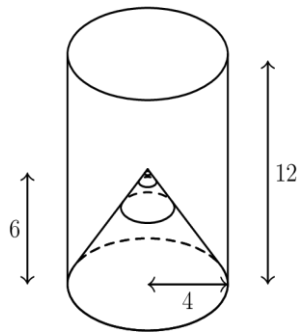


Fig.1

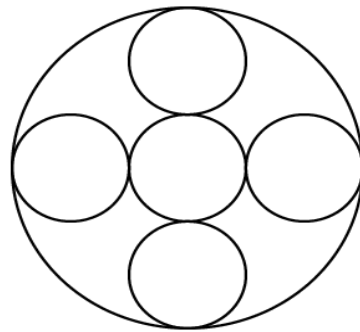


Fig. 2